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Radiative effects of fourth-rank SU(N) tensor Higgs fields

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Abstract. We investigate the destabilising influence of the self-dual fourth-rank tensor scalar fields on the asymptotic freedom in standard SU(N) gauge theories. In particular, we consider the special cases SU(2), SU(3), SU(4) and SU(5) with 5-, 27-, 20- and 75-dimensional scalar fields, respectively. We stress the interesting phenomenological application of our equations to the model of broken QCD with 27 scalars.

1. Introduction

The problem of incorporating scalars in asymptotically free theories was first examined by Gross and Wilczek [1]. The effect of the presence of Yukawa couplings on the asymptotic freedom of the theory was studied by Cheng *et al* [2] and they also investigated the asymptotic stability of the scalar coupling constants in more complicated cases of reducible representations of scalar fields for SU(N) and O(N) groups. However, for their purposes and for simplicity, they restricted themselves to cases containing at most second-rank tensors.

In constructing unified models there is no principal reason to restrict representations of scalar fields to low-rank tensors only. Indeed, there exist in the literature a number of different GUT and SUSY GUT models using higher rank tensors [3, 4].

Furthermore, in order to account for possibly free fractionally charged particles, Slansky *et al* [5] have proposed that QCD may be broken to a SO(3) subgroup, with SU(3) triplets becoming SO(3) triplets. The breaking can be done with a 27-dimensional scalar field. It is interesting to investigate all consequences of such a model: Glück and Reya [6] have shown that very light coloured scalars preserve all presently observed short-distance properties of QCD, even for large multiplets of SU(3). However, it is expected that a gauge theory containing an elementary scalar field in higher representation is not asymptotically free, and therefore the question naturally arises, to which extent the evolution of the gauge coupling is disturbed by the destabilising influence of quartic couplings λ_i. Nevertheless it is also interesting to investigate, on the basis of a leading order renormalisation group analysis, whether this model of broken QCD can be temporarily free, and thus still compatible with present experiments, as was done for some other models of broken QCD [7]. To follow this programme we are motivated to construct and study those leading order renormalisation group equations (LO RGE) which are needed for studying the asymptotical behaviour of (broken) QCD with a 27-dimensional scalar field.

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The related problem of finding the minima of SU(N) invariant potentials with self-dual fourth-rank tensor is very complex and has been solved only partially [8, 9, 12]. Abud *et al* [9] have recently found, studying minima of a SU(5) invariant Higgs potential with one real 75 representation, a counterexample to Michel's conjecture [10], which reduces possible minima to a class of maximal little (isotropy) groups. However, this counterexample is a consequence of a very special relation between quartic coupling constants, which is not protected by any kind of symmetry. In the case with additional Z_2 symmetry Cummins and King [12] have found stable absolute minimum invariant under non-maximal isotropy group. Studying the SU(5) model with 75 in more detail it is useful to know appropriate RGE, i.e. one-loop radiative Higgs potential.

The present paper is organised as follows. In § 2 we present the most general potential for self-dual fourth-rank tensors of SU(N) groups which is given by six independent quartic and two cubic invariants. In § 3 we repeat the general formulae for deriving the appropriate coefficients in LO RGE, specifying them for our case in which Yukawa couplings are not present. In § 4 we write explicitly LO RGE for four cases: SU(2), SU(3) with symmetric tensors (5 and 27, respectively) and SU(4), SU(5) with antisymmetric tensors (20 and 75, respectively). Our conclusions are drawn in § 5. In appendix 1 we give 60 coefficients which enter the RGE for the general case and which are sufficient to construct our four special cases. They also provide a consistency check of our calculations. In appendix 2 we present all relations between quartic invariants for the above four cases and explain their group theoretical origin.

2. Potential and invariants for scalar fields

We consider scalar fields transforming as a self-dual fourth-rank tensor under SU(N): $T^{\alpha\beta}_{\gamma\delta}$, $\alpha, \beta, \gamma, \delta = 1, \ldots, N$ which satisfies the symmetry conditions

$$T^{\alpha\beta}_{\gamma\delta} = \pi T^{\beta\alpha}_{\gamma\delta} = \pi T^{\alpha\beta}_{\delta\gamma} \tag{2.1}$$

where $\pi = +1$ corresponds to a tensor symmetric in upper and lower indices; and $\pi = -1$ corresponds to a tensor antisymmetric in upper and lower indices; and the tracelessness and reality conditions, respectively, are

$$T^{\alpha\beta}_{\gamma\beta} = 0 \qquad (T^{\alpha\beta}_{\gamma\delta})^* = T^{\gamma\delta}_{\alpha\beta}. \tag{2.2}$$

This tensor represents an irreducible representation of SU(N) with dimension

$$d(T) = \tfrac{1}{4}N^2(N-\pi)(N+3\pi). \tag{2.3}$$

The most general renormalisable SU(N) invariant potential [8] is given in terms of six quartic, two cubic and one quadratic invariants

$$V(T) = m^2 Q + c_1 C_1 + c_2 C_2 + \tfrac{1}{2}\lambda_0 I_0 + \sum_{i=1}^{5} \lambda_i I_i \tag{2.4}$$

where

$$\begin{aligned}
&Q = T^{\alpha\beta}_{\gamma\delta} T^{\gamma\delta}_{\alpha\beta} \\
&C_1 = T^{\alpha\beta}_{\gamma\delta} T^{\gamma\delta}_{\mu\nu} T^{\mu\nu}_{\alpha\beta} && C_2 = T^{\alpha\beta}_{\gamma\delta} T^{\gamma\mu}_{\beta\nu} T^{\nu\delta}_{\mu\alpha} \\
&I_1 = T^{\alpha\beta}_{\gamma\delta} T^{\gamma\delta}_{\mu\nu} T^{\mu\nu}_{\lambda\rho} T^{\lambda\rho}_{\alpha\beta} && I_2 = T^{\alpha\beta}_{\gamma\delta} T^{\delta\mu}_{\beta\nu} T^{\nu\lambda}_{\mu\rho} T^{\rho\gamma}_{\lambda\alpha} \\
&I_3 = T^{\alpha\beta}_{\gamma\delta} T^{\gamma\delta}_{\mu\nu} T^{\mu\lambda}_{\beta\rho} T^{\rho\nu}_{\lambda\alpha} && I_4 = T^{\alpha\beta}_{\gamma\delta} T^{\delta\mu}_{\beta\nu} T^{\nu\lambda}_{\alpha\rho} T^{\rho\gamma}_{\lambda\mu} \\
&I_5 = T^{\alpha\beta}_{\gamma\delta} T^{\gamma\delta}_{\mu\beta} T^{\mu\nu}_{\lambda\rho} T^{\lambda\rho}_{\alpha\nu} && I_0 = Q^2.
\end{aligned} \tag{2.5}$$

These invariants are independent [8, 11] for $\pi=+1$, when $N \geqslant 4$, and for $\pi=-1$ when $N \geqslant 8$. The relations between quartic invariants are explicitly given in appendix 2.

Though the tensor notations are very convenient for constructing invariants under SU(N), they are very clumsy for working out the Feynman rules, because four indices are required to label these fields. Thus we make a transformation to a one index labelling by writing

$$T^{\alpha\beta}_{\gamma\delta} = (B^i)^{\alpha\beta}_{\gamma\delta}\phi^i \qquad i=1,\ldots,d(T) \tag{2.6}$$

where (B^i) is a set of traceless tensors with the same symmetry properties as T. The B^i are normalised in the following way:

$$\mathrm{Tr}(B^iB^j) = (B^i)^{\alpha\beta}_{\gamma\delta}(B^j)^{\gamma\delta}_{\alpha\beta} = \tfrac{1}{2}\delta_{ij} \tag{2.7}$$

so we can express the fields ϕ^i as

$$\phi^i = 2(B^i)^{\alpha\beta}_{\gamma\delta}T^{\gamma\delta}_{\alpha\beta}. \tag{2.8}$$

The B^i form a complete set of tensors with completeness relation

$$\sum_{i=1}^{d(T)}(B^i)^{\alpha\beta}_{\gamma\delta}(B^i)^{\mu\nu}_{\lambda\rho} = \tfrac{1}{8}(\delta^{\alpha\beta}_{\lambda\rho}+\pi\delta^{\alpha\beta}_{\rho\lambda})(\delta^{\mu\nu}_{\gamma\delta}+\pi\delta^{\mu\nu}_{\delta\gamma}) - (B_0)^{\alpha\beta}_{\gamma\delta}(B_0)^{\mu\nu}_{\lambda\rho}$$
$$-\sum_{a=1}^{N^2-1}(B_a)^{\alpha\beta}_{\gamma\delta}(B_a)^{\mu\nu}_{\lambda\rho} \tag{2.9}$$

where

$$\begin{aligned}(B_0)^{\alpha\beta}_{\gamma\delta} &= [4N(N+\pi)]^{-1/2}(\delta^{\alpha\beta}_{\gamma\delta}+\pi\delta^{\alpha\beta}_{\delta\gamma})\\ (B_a)^{\alpha\beta}_{\gamma\delta} &= [4(N+2\pi)^{-1/2}][\delta^\alpha_\gamma(t^a)^\beta_\delta+\pi\delta^\beta_\gamma(t^a)^\alpha_\delta+\pi\delta^\alpha_\delta(t^a)^\beta_\gamma+\delta^\beta_\delta(t^a)^\alpha_\gamma]\end{aligned} \tag{2.10}$$

and $(t^a)^\alpha_\beta$ ($a=1,\ldots,N^2-1$) are generators in the fundamental representation.

Now we can write the potential in terms of the fields ϕ_i

$$V(\phi) = \frac{1}{4!}f_{ijkl}\phi_i\phi_j\phi_k\phi_l + \frac{1}{3!}c_{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}m^2\phi_i\phi_i \tag{2.11}$$

where f_{ijkl}, c_{ijk} are totally symmetric.

Using (2.13) and (2.14) we find the most general quartic scalar coupling vertex

$$\begin{aligned}f_{ijkl} = {}& \lambda_0(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) + 4\lambda_1((ijkl)_1+(iklj)_1+(ikjl)_1+(k\leftrightarrow l)]\\ &+8\lambda_2((ijkl)_2+(ijlk)_2+(ikjl)_2)\\ &+2\lambda_3\{[(ijkl)_3+(ikjl)_3+(iljk)_3+(klij)_3+(jlik)_3+(jkil)_3]+\mathrm{cc}\}\\ &+4\lambda_4\{[(ijkl)_4+(ikjl)_4+(iljk)_4+\mathrm{cc}\}\\ &+2\lambda_5\{[(ijkl)_5+(iklj)_5+(ikjl)_5)+(k\leftrightarrow l)]+\mathrm{cc}\}\end{aligned} \tag{2.12}$$

where quartic quantities $(ijkl)_J$, $J=0,\ldots,5$, are defined in terms of invariants (2.5), as

$$I_J = (ijkl)_J\phi_i\phi_j\phi_k\phi_l \tag{2.13}$$

with the following properties

$$
\begin{aligned}
&(ijkl)_0=(ij)(kl)=\tfrac{1}{4}\delta_{ij}\delta_{kl}\\
&(ijkl)_1=(jkli)_1=(lkji)_1^*=\text{cyclic permutation}\\
&(ijkl)_2=(jkli)_2=(lkji)_2=\text{cyclic permutation}\\
&(ijkl)_3=(ijlk)_3=(jikl)_3^*\\
&(ijkl)_4=(jilk)_4=(klji)_4=(ijlk)_4^*\\
&(ijkl)_5=(klij)_5=(jilk)_5^*.
\end{aligned}
\tag{2.14}
$$

The quartic coupling vertex (2.12) can be written symbolically

$$
\begin{aligned}
f_{ijkl}=4\lambda_0[(ij)(kl)]^{S_3}+4\lambda_1[(ijkl)_1]^{S_6}+8\lambda_2[(ijkl)_2]^{S_3}&\\
+2\lambda_3[(ijkl)_3]^{S_{12}}+4\lambda_4[(ijkl)_4]^{S_6}+2\lambda_5[(ijkl)_5]^{S_{12}}&
\end{aligned}
\tag{2.15}
$$

where S_i denotes the sum of i appropriate terms.

For completeness we also give the cubic scalar vertex

$$c_{ijk}=3c_1[(ijk)_1+(jik)_1]+6c_2(ijk)_2 \tag{2.16}$$

where cubic quantities $(ijk)_{1,2}$ are defined as

$$C_{1,2}=(ijk)_{1,2}\phi_i\phi_j\phi_k. \tag{2.17}$$

$(ijk)_1$ is invariant under cyclic permutation, and $(ijk)_2$ is invariant under all permutations of i, j, k indices.

3. Renormalisation group equations

First of all we briefly repeat all starting points necessary for our calculation. Here we are interested only in those differential equations for the coupling constants which are needed for studying whether gauge theories can be asymptotically (or at least temporarily) free in the presence of the Higgs phenomenon.

Let A_μ^a, ϕ_i and ψ_α be the Hermitian gauge fields, real scalar fields and spin-$\frac{1}{2}$ fields, respectively. Here we restrict ourselves to scalar fields ϕ_i which transform as a self-dual fourth-rank tensor, and to spin-$\frac{1}{2}$ fields which transform as a reducible representation consisting of N_f fundamental (vector) representations of the SU(N) group. So the most general renormalisable gauge invariant Lagrangian is

$$\mathscr{L}=-\tfrac{1}{4}F_{\mu\nu}^aF^{a,\mu\nu}+\tfrac{1}{2}(D_\mu\phi)_i(D^\mu\phi)_i-V(\phi)+\bar{\psi}(\mathrm{i}\gamma^\mu D_\mu-m_0)\psi \tag{3.1}$$

where the quartic potential $V(\phi)$ is given by (2.11) and (2.12), and where

$$
\begin{aligned}
&F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+gC^{abc}A_\mu^bA_\nu^c\\
&(D_\mu\phi)_i=\partial_\mu\phi_i-\mathrm{i}g(t_\mathrm{S}^a)_{ij}\phi_jA_\mu^a\\
&(D_\mu\psi)_\alpha=\partial_\mu\psi_\alpha-\mathrm{i}g(t_\mathrm{F}^a)_{\alpha\beta}\psi_\beta A_\mu^a.
\end{aligned}
\tag{3.2}
$$

C^{abc} are SU(N) structure constants and $(t_\mathrm{S}^a)_{ij}$, $(t_\mathrm{F}^a)_{\alpha\beta}$ are representation matrices of the generators in case of scalar and fermion fields, respectively.

Note that in our Lagrangian (3.1) there are no Yukawa coupling terms, since the product of two fundamental representations contains at most second-rank tensor representations.

Now specifying the Lagrangian, we write the lowest-order approximation of the renormalisation group equations for the gauge and quartic coupling constants, respectively

$$\frac{dg}{dt}=-\frac{1}{16\pi^2}[\tfrac{11}{3}N-\tfrac{4}{3}S_2(F)-\tfrac{1}{6}S_2(S)]g^3\equiv-\frac{1}{32\pi^2}b_0g^3 \tag{3.3}$$

$$\frac{df_{ijkl}}{dt}=\frac{1}{16\pi^2}(f_{ijmn}f_{mnkl}+f_{ikmn}f_{mnjl}+f_{ilmn}f_{mnjk}-12C^{(2)}(S)g^2f_{ijkl}+3A_{ijkl}g^4) \tag{3.4}$$

where

$$A_{ijkl}=\{t^a_S,t^b_S\}_{ij}\{t^a_S,t^b_S\}_{kl}+\{t^a_S,t^b_S\}_{ik}\{t^a_S,t^b_S\}_{jl}+\{t^a_S,t^b_S\}_{il}\{t^a_S,r^b_S\}_{jk}. \tag{3.5}$$

Here we have used the following identities

$$\begin{aligned}&(t^a_Rt^a_R)=C^{(2)}(R)\mathbb{1}\\&\mathrm{Tr}(t^a_Rt^b_R)=S_2(R)\delta_{ab}\\&d(R)C^{(2)}(R)=(N^2-1)S_2(R)\end{aligned} \tag{3.6}$$

where the index R denotes the representation.

For fermions one has $S_2(F)=\frac{1}{2}N_f$, whereas the scalar fields transforming as a self-dual fourth-rank π symmetric tensor of SU(N) satisfy

$$C^{(2)}(T)=2(N+\pi)\qquad S_2(T)=\tfrac{1}{2}N^2(N+3\pi). \tag{3.7}$$

From (3.3) we find that for $\pi=+1$, $N\geqslant 6$ and $\pi=-1$, $N\geqslant 9$, b_0 becomes negative. Actually the contribution of the scalar field is so large that asymptotic freedom does not hold, even when $N_f\leqslant 2$ and $N_f\leqslant 4$, for $\pi=+1$ and $\pi=-1$, respectively.

The representation matrices of generators acting on scalar fields (2.1) are given by

$$\begin{aligned}&(t^a_S)_{ij}=4(B^i)^{\gamma\delta}_{\alpha\beta}[(t^a)^\alpha_\mu(B^j)^{\mu\beta}_{\gamma\delta}-(t^a)^\mu_\gamma(B^j)^{\alpha\beta}_{\mu\delta}]\\&(t^a_S)_{ij}=-(t^a_S)_{ji}\end{aligned} \tag{3.8}$$

where the B^i tensors are defined in (2.6) and (2.7), and $(t^a)^\alpha_\beta$ are generators in the fundamental representation. Using the explicit expression (3.8) we find

$$A_{ijkl}=16\{2[(ij)(kl)]^{S_3}+2[(ijkl)_1]^{S_6}+16[(ijkl)_2]^{S_3}-4[(ijkl)_3]^{S_{12}}+\tfrac{1}{2}(N+4\pi)[(ijkl)_5]^{S_{12}}\} \tag{3.9}$$

where we have used the same notation as for f_{ijkl} in (2.15).

Since the quartic scalar vertex (2.12) and (2.15) is rather complicated, the calculation of appropriate quadratic combinations in (3.4) is very tedious and lengthy. These contributions correspond to one-loop vertex corrections containing scalar fields only. The general result of this calculation has the following form:

$$f_{ijmn}f_{mnkl}+f_{ikmn}f_{mnjl}+f_{ilmn}f_{mnjk}=16\left(\sum_q\lambda_q^2\sum_I a^I_{qq}[(ijkl)_I]^{S_I}+\sum_{q<r}\lambda_q\lambda_r\sum_I a^I_{qr}[(ijkl)_I]^{S_I}\right) \tag{3.10}$$

where $q, r, I=0,\ldots,5$ and S_I denotes symmetrisation of $(ijkl)_I$ term, as defined in (2.15). There are 126 a^I_{qr} coefficients to be calculated. In appendix 1 we give only 60 of them which are sufficient to construct RGE for our four special cases, as well as to allow for the consistency check of our calculation.

We note that the one-loop radiative correction to the Higgs potential, due to the gauge boson, is given by $3/64\pi^4 \,\mathrm{Tr}\, M_V^4 \ln M_V^2$, where M_V^2 is the gauge boson mass matrix. Thus the initial rolling direction of the vacuum, when released from the initial symmetric phase, can be found by maximising $\mathrm{Tr}\, M_V^4/Q^2$. From (2.15) and (3.9) we find

$$\mathrm{Tr}\, M_V^4 = 16g^4[Q^2 + 2I_1 + 8I_2 - 8I_3 + (N+4\pi)I_5]. \tag{3.11}$$

4. Special cases

There are only a few cases, namely $\pi = +1$, $N \leq 5$ and $\pi = -1$, $N \leq 8$, in which the gauge couplings are still asymptotically free (in the lowest order approximation). Since the interesting phenomenological application of the RG equations (3.4) is for SU(3), which has four independent quartic invariants, we restrict ourselves to all cases which can be described by at most four independent quartic invariants. We choose I_0, I_1, I_2 and I_5.

To construct appropriate RG equations, it is necessary to know the exact relations between quartic invariants [11] which are given in appendix 2. So we present here only four special cases for $\pi = +1$: SU(2), SU(3), and for $\pi = -1$: SU(4), SU(5).

(i) *SU(2), $\pi = +1$, $d(T) = 5$*

Starting from (2.4) and (A2.1), we find for the quartic part of the potential

$$V(\phi) = \tfrac{1}{2}(\lambda_0 + \lambda_1 + \lambda_2 + \lambda_5)I_0 \equiv \tfrac{1}{2}\lambda_0' I_0. \tag{4.1}$$

Using (3.4), (3.7)-(3.10), (4.1) and appendix 1 we find the LO RGE for the scalar quartic coupling

$$\frac{\mathrm{d}\lambda_0'}{\mathrm{d}t} = \frac{1}{4\pi^2}\left(\tfrac{13}{4}\lambda_0'^2 - 18g^2\lambda_0' + 54g^4\right). \tag{4.2}$$

In terms of the new variable $\bar\lambda = \lambda_0'/g^2$, (4.2) becomes

$$\frac{1}{g^2}\frac{\mathrm{d}\bar\lambda}{\mathrm{d}t} = \frac{1}{4\pi^2}\left(\tfrac{13}{4}\bar\lambda^2 + (\tfrac{1}{4}b_0 - 18)\bar\lambda + 54\right) \tag{4.3}$$

where b_0 is defined by

$$\frac{\mathrm{d}g^2}{\mathrm{d}t} = -\frac{b_0}{16\pi^2}g^4 \qquad b_0 = \tfrac{2}{3}(17 - 2N_f). \tag{4.4}$$

It is obvious from (4.3) that for $\bar\lambda$ exist no ultraviolet stable fixed point, and so asymptotic freedom cannot be maintained for this case.

(ii) *SU(3), $\pi = +1$, $d(T) = 27$*

Using (2.4) and (A2.2), the quartic part of the potential can be written

$$\begin{aligned} V(\phi) &= \tfrac{1}{2}(\lambda_0 + \tfrac{1}{4}\lambda_3 + \lambda_4)I_0 + (\lambda_1 - \tfrac{1}{4}\lambda_3)I_1 + (\lambda_2 + \lambda_4)I_2 + (\lambda_5 - 2\lambda_4)I_5 \\ &= \tfrac{1}{2}\lambda_0' I_0 + \lambda_1' I_1 + \lambda_2' I_2 + \lambda_5' I_5. \end{aligned} \tag{4.5}$$

Using (3.4), (3.7)-(3.10), (4.5) and the results of appendix 1 we find the LO RGE for scalar quartic couplings

$$\frac{d\lambda_0'}{dt}=\frac{1}{4\pi^2}\left(\frac{35}{4}\lambda_0'^2+\frac{91}{10}\lambda_0'\lambda_1'+\frac{277}{40}\lambda_0'\lambda_2'+\frac{53}{5}\lambda_0'\lambda_5'+\frac{357}{100}\lambda_1'^2\right.$$
$$\left.-\frac{67}{50}\lambda_1'\lambda_2'+\frac{97}{25}\lambda_1'\lambda_5'-\frac{303}{1600}\lambda_2'^2+\frac{53}{25}\lambda_2'\lambda_5'+\frac{521}{200}\lambda_5'^2-24g^2\lambda_0'+0\right)$$

$$\frac{d\lambda_1'}{dt}=\frac{1}{4\pi^2}\left(3\lambda_0'\lambda_1'+\frac{193}{50}\lambda_1'^2+\frac{117}{200}\lambda_1'\lambda_2'+\frac{48}{25}\lambda_1'\lambda_5'+\frac{3}{100}\lambda_2'^2\right.$$
$$\left.+\frac{19}{100}\lambda_2'\lambda_5'+\frac{1}{20}\lambda_5'^2-24g^2\lambda_1'+12g^4\right)$$

$$\frac{d\lambda_2'}{dt}=\frac{1}{4\pi^2}\left(3\lambda_0'\lambda_2'+\frac{8}{25}\lambda_1'^2+\frac{1}{5}\lambda_1'\lambda_2'+\frac{8}{25}\lambda_1'\lambda_5'+\frac{89}{40}\lambda_2'^2\right.$$
$$\left.+\frac{14}{5}\lambda_2'\lambda_5'+\frac{9}{50}\lambda_5'^2-24g^2\lambda_2'+24g^4\right)$$

$$\frac{d\lambda_5'}{dt}=\frac{1}{4\pi^2}\left(3\lambda_0'\lambda_5'-2\lambda_1'^2+\frac{229}{25}\lambda_1'\lambda_2'+\frac{109}{25}\lambda_1'\lambda_5'+\frac{51}{20}\lambda_2'^2\right.$$
$$\left.+\frac{219}{50}\lambda_2'\lambda_5'+\frac{293}{100}\lambda_5'^2-24g^2\lambda_5'+21g^4\right) \quad (4.6)$$

and the LO RGE for the gauge coupling constant (3.3):

$$\frac{dg}{dt}=-\frac{b_0}{32\pi^2}g^3 \qquad b_0=13-\tfrac{4}{3}N_f \quad (4.7)$$

with $b_0>0$ for $N_f\leqslant 9$.

Using the new set of variables $\bar{\lambda}_i=\lambda_i/g^2$ and analysing the transformed equations, we find that asymptotic freedom cannot be maintained for this case. In fact this was expected, since even in the case of SU(3) gauge theory containing one scalar field in the adjoint representation, asymptotic freedom is lost.

(iii) *SU(4),* $\pi=-1$, $d(T)=20$

Using (2.4) and (A2.3), the quartic part of the potential can be written

$$V(\phi)=\tfrac{1}{2}(\lambda_0+\tfrac{3}{8}\lambda_2+\tfrac{1}{4}\lambda_3+\tfrac{1}{4}\lambda_4+\tfrac{1}{2}\lambda_5)I_0+(\lambda_1-\tfrac{1}{8}\lambda_2-\tfrac{1}{4}\lambda_3-\tfrac{1}{4}\lambda_4)I_1=\tfrac{1}{2}\lambda_0'I_0+\lambda_1'I_1. \quad (4.8)$$

Using (3.4), (3.7)-(3.10), (4.8) and appendix 1 we find LO RGE for scalar quartic and gauge couplings

$$\frac{d\lambda_0'}{dt}=\frac{1}{4\pi^2}(7\lambda_0'^2+\tfrac{7}{4}\lambda_1'^2+7\lambda_0'\lambda_1'-18g^2\lambda_0'+9g^4)$$
$$\frac{d\lambda_1'}{dt}=\frac{1}{4\pi^2}(\tfrac{15}{4}\lambda_1'^2+3\lambda_0'\lambda_1'-18g^2\lambda_1'+9g^4) \quad (4.9)$$

$$\frac{dg}{dt}=-\frac{b_0}{32\pi^2}g^3 \qquad b_0=\tfrac{2}{3}(40-2N_f) \quad (4.10)$$

with $b_0>0$ for $N_f<20$.

(iv) $SU(5)$, $\pi=-1$, $d(T)=75$

Using (2.4) and (A2.4), the quartic part of the potential can be written

$$V(\phi)=\tfrac{1}{2}(\lambda_0-\tfrac{5}{8}\lambda_2-\tfrac{1}{4}\lambda_3-\tfrac{1}{2}\lambda_4)I_0+(\lambda_1-\tfrac{1}{8}\lambda_2-\tfrac{1}{4}\lambda_3-\tfrac{1}{4}\lambda_4)I_1+(2\lambda_2+\lambda_3+\tfrac{3}{2}\lambda_4+\lambda_5)I_5$$
$$=\tfrac{1}{2}\lambda'_0I_0+\lambda'_1I_1+\lambda'_5I_5. \tag{4.11}$$

Using equations (3.4), (3.7)-(3.10), (4.11) and appendix 1 we find the LO RGE for the scalar quartic couplings

$$\frac{d\lambda'_0}{dt}=\frac{1}{4\pi^2}(\tfrac{83}{4}\lambda_0'^2+\tfrac{91}{6}\lambda'_0\lambda'_1+\tfrac{49}{3}\lambda'_0\lambda'_5+\tfrac{113}{36}\lambda_1'^2+\tfrac{49}{9}\lambda'_1\lambda'_5+\tfrac{143}{48}\lambda_5'^2-24g^2\lambda'_0-3g^4)$$

$$\frac{d\lambda'_1}{dt}=\frac{1}{4\pi^2}(3\lambda'_0\lambda'_1+\tfrac{13}{2}\lambda_1'^2+2\lambda'_1\lambda'_5-\tfrac{1}{144}\lambda_5'^2-24g^2\lambda'_1+9g^4) \tag{4.12}$$

$$\frac{d\lambda'_5}{dt}=\frac{1}{4\pi^2}(3\lambda'_0\lambda'_5-2\lambda_1'^2+\tfrac{7}{3}\lambda'_1\lambda'_5+\tfrac{17}{9}\lambda_5'^2-24g^2\lambda'_5+27g^4)$$

and the LO RGE for the gauge coupling constant (3.3) becomes

$$\frac{dg}{dt}=-\frac{b_0}{32\pi^2}g^3 \qquad b_0=\tfrac{1}{3}(85-4N_f) \tag{4.13}$$

with $b_0>0$ for $N_f\leqslant 21$.

It can also be shown numerically that cases (iii) and (iv) are not asymptotically free.

5. Conclusion

LO RGE for the quartic scalar couplings have been constructed in standard SU(N) gauge theories containing fermions in N_f fundamental representations and scalar fields in self-dual fourth-rank (symmetric and antisymmetric) tensor representations. These RGE are needed for studying whether the gauge theories can be asymptotically or temporarily free in the presence of the Higgs phenomenon.

We have found that for $\pi=+1$, $N\geqslant 6$ and $\pi=-1$, $N\geqslant 9$, the contribution of the scalar field to the gauge coupling RGE destabilises asymptotic freedom, even when there are $N_f\leqslant 2$, i.e. $N_f\leqslant 4$ fermions, respectively.

In order to investigate the physically interesting case of broken QCD with 27 scalars, we have restricted ourselves to those special cases where the most general quartic Higgs potential can be described in terms of maximally four independent quartic invariants. These cases are: SU(2), SU(3), SU(4) and SU(5) with 5-, 27-, 20- and 75-dimensional representations of scalars, respectively. For SU(2) it is obvious from (4.3) that asymptotic freedom cannot be maintained. It can be shown numerically that the same result holds for the remaining three cases, as is expected.

Finally we note that it is interesting to perform a detailed phenomenological analysis for broken QCD with 27 scalars using our RGE (4.6), as well as all conditions which ensure SO(3) to be an absolute minimum. This analysis is necessary to test whether this model of broken QCD is still compatible with present experiments. Results of this investigation will be presented elsewhere.

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Appendix 1

Here we give the results of our calculation for 60 a^I_{qr} coefficients, which enter (3.4) and (3.10). The indices q and r are restricted to 0, 1, 2, 5. (In the calculation we have used completeness relation (2.9).)

$$a^0_{00}=\tfrac{1}{4}d(T)+2 \qquad a^1_{00}=a^2_{00}=a^3_{00}=a^4_{00}=a^5_{00}=0$$

$$a^0_{01}=N(N+\pi)-\frac{4N}{N+\pi}+\frac{2}{(N+\pi)(N+2\pi)} \qquad a^1_{01}=3 \qquad a^2_{01}=a^3_{01}=a^4_{01}=a^5_{01}=0$$

$$a^0_{02}=\frac{1}{2}\left((N+\pi)^2-\frac{5}{2}+\frac{\pi N+4}{(N+\pi)(N+2\pi)}\right) \qquad a^2_{02}=6 \qquad a^1_{02}=a^3_{02}=a^4_{02}=a^5_{02}=0$$

$$a^0_{05}=\frac{d(T)}{N}+\tfrac{1}{2}(N+\pi)-\frac{2}{N+2\pi} \qquad a^5_{05}=\tfrac{3}{2} \qquad a^1_{05}=a^2_{05}=a^3_{05}=a^4_{05}=0$$

$$a^0_{11}=3+\frac{8(N+\pi)^2+36}{(N+\pi)^2(N+2\pi)^2}$$

$$a^1_{11}=\frac{N(N+\pi)}{2}-\frac{4N}{N+\pi}+\frac{8}{(N+2\pi)^2}+\frac{14}{(N+\pi)(N+2\pi)}$$

$$a^2_{11}=\frac{16}{(N+2\pi)^2} \qquad a^3_{11}=\frac{8}{(N+2\pi)^2} \qquad a^4_{11}=0 \qquad a^5_{11}=-\frac{4(N^2+2\pi N+10)}{(N+\pi)(N+2\pi)^2}$$

$$a^0_{12}=\frac{4(N+\pi)^2+18}{(N+\pi)^2(N+2\pi)^2} \qquad a^1_{12}=\frac{2}{(N+2\pi)^2} \qquad a^2_{12}=\frac{12}{(N+2\pi)^2}+2$$

$$a^3_{12}=-\frac{3}{2}+\frac{\pi(22+12\pi N-N^2)}{(N+\pi)(N+2\pi)^2} \qquad a^4_{12}=-\frac{2(3N+4\pi)}{(N+2\pi)^2}$$

$$a^5_{12}=\frac{N(N+3\pi)}{N+2\pi}+\frac{3\pi(N-5\pi)}{(N+\pi)(N+2\pi)^2}$$

$$a^0_{15}=\frac{N(N+3\pi)(N^2+3\pi N+6)-4}{(N+\pi)(N+2\pi)^2} \qquad a^1_{15}=\frac{N+\pi}{2}-\frac{4}{N+2\pi} \qquad a^2_{15}=0$$

$$a^3_{15}=-\frac{4(2N+3\pi)}{(N+2\pi)^2} \qquad a^4_{15}=\frac{8}{(N+2\pi)^2}$$

$$a^5_{15}=1+\frac{N(N+3\pi)(N^2+2\pi N-8)+24}{4(N+2\pi)^2}$$

$$a^0_{22}=\frac{3}{8}+\frac{2(N+\pi)^2+9}{4(N+\pi)^2(N+2\pi)^2} \qquad a^1_{22}=\frac{1}{16}+\frac{1}{8(N+2\pi)^2}$$

$$a_{22}^2 = \frac{(N+2\pi)^2}{4} - \frac{1}{2} + \frac{\pi(33+7\pi N - 8N^2)}{4(N+\pi)(N+2\pi)^2} \qquad a_{22}^3 = \frac{1}{8}\left(1 - \frac{1+3\pi N}{(N+2\pi)^2}\right)$$

$$a_{22}^4 = -\frac{\pi}{2} - \frac{3(2N^2+\pi N-4)}{4(N+\pi)(N+2\pi)^2} \qquad a_{22}^5 = \frac{N+3\pi}{8} - \frac{1}{2(N+2\pi)} + \frac{2\pi N - 7}{4(N+\pi)(N+2\pi)^2}$$

$$a_{25}^0 = \frac{1}{2}\left(N+\pi+\frac{\pi N - 5}{(N+\pi)(N+2\pi)^2}\right) \qquad a_{25}^1 = 0 \qquad a_{25}^2 = N+3\pi - \frac{2(3N+4\pi)}{(N+2\pi)^2}$$

$$a_{25}^3 = -\frac{4N+7\pi}{2(N+2\pi)^2} \qquad a_{25}^4 = \frac{N^2-1}{(N+2\pi)^2} \qquad a_{25}^5 = \frac{1}{4} + \frac{N(N+\pi)(N+3\pi)^2+20}{8(N+2\pi)^2}$$

$$a_{55}^0 = \frac{N(N+3\pi)(3N+2\pi)}{16(N+2\pi)} + \frac{3}{8} + \frac{2}{(N+2\pi)^2} \qquad a_{55}^1 = \frac{1}{8} \qquad a_{55}^2 = 1 + \frac{4}{(N+2\pi)^2}$$

$$a_{55}^3 = \frac{N}{4(N+2\pi)} \qquad a_{55}^4 = -\frac{2}{N+2\pi}$$

$$a_{55}^5 = \frac{N(N+\pi)(N+2\pi)}{64} + \frac{N+4\pi}{16} - \frac{7N+10\pi}{4(N+2\pi)^2}.$$

We note the regularity of these expressions, namely that all of them can be written as ratios of homogeneous polynomials in N and π (with $\pi^2=1$). We have also checked the self-consistency of *all* of the above coefficients; namely for cases with less than four independent quartic invariants, there exists a number of constraints on the above presented coefficients. Using the relations between invariants (given in appendix 2), one can easily check that for (SU(2), $\pi=+1$), SU(4), $\pi=-1$) and (SU(5), $\pi=-1$) there exist 9, 14 and 12 constraints, respectively, which are satisfied.

The remaining 66 coefficients in (3.10) are necessary for constructing the RGE (3.4), when $N \geqslant 4$, $\pi=+1$ and $N \geqslant 6$, $\pi=-1$. Since the appropriate equations are not of immediate physical interest, and since the required calculations are very lengthy, we have omitted them.

Appendix 2

Here we present the relations between quartic invariants [11] for the four cases of special interest:

(i) SU(2) $\pi=+1$ $d(T)=5$

$$I_0 = 2I_1 = 2I_2 = 2I_5 \qquad I_3 = I_4 = 0. \tag{A2.1}$$

(ii) SU(3) $\pi=+1$ $d(T)=27$

$$I_3 = \tfrac{1}{8}I_0 - \tfrac{1}{4}I_1 \qquad I_4 = \tfrac{1}{2}I_0 + I_2 - 2I_5. \tag{A2.2}$$

(iii) SU(4) $\pi=-1$ $d(T)=20$

$$I_2 = \tfrac{1}{16}(3I_0 - 2I_1) \qquad I_3 = I_4 = \tfrac{1}{8}(I_0 - 2I_1) \qquad I_5 = \tfrac{1}{4}I_0. \tag{A2.3}$$

(iv) SU(5) $\pi=-1$ $d(T)=75$

$$I_2 = -\tfrac{5}{16}I_0 - \tfrac{1}{8}I_1 + 2I_5 \qquad I_3 = -\tfrac{1}{8}I_0 - \tfrac{1}{4}I_1 + I_5 \qquad I_4 = -\tfrac{1}{4}I_0 - \tfrac{1}{4}I_1 + \tfrac{3}{2}I_5. \tag{A2.4}$$

The group theoretical origin of these relations is as follows:

(a) There are two bilinear combinations of the initial tensor T

$$(T^{\alpha\beta}_{\mu\gamma}T^{\mu\nu}_{\gamma\delta} - \text{traces})$$

and (A2.5)

$$(T^{\alpha\mu}_{\gamma\nu}T^{\beta\nu}_{\delta\mu} + \pi T^{\beta\mu}_{\gamma\nu}T^{\alpha\nu}_{\delta\mu} - \text{traces})$$

which transform in the same way as T (with π symmetry). They are linearly dependent if the *symmetric* part of the Kronecker product, $(\pi\times\pi)_{\text{symm}}$, contains one π representation only. An appropriate relation between these two bilinear combinations (A2.5), results in one relation between cubic invariants and two relations between quartic invariants. This happens for SU(2) with $\pi=+1$ and for SU(4) and SU(5) with $\pi=-1$.

(b) There is also one bilinear combination:

$$(T^{\alpha\mu}_{\gamma\nu}T^{\beta\nu}_{\delta\mu} - \pi T^{\beta\mu}_{\gamma\nu}T^{\alpha\nu}_{\delta\mu} - \text{traces}) \tag{A2.6}$$

which transforms as self-dual fourth-rank tensor, but with opposite symmetry $(-\pi)$. If this representation does not appear in the symmetric part of the $(\pi\times\pi)$ Kronecker product (or if it is a trivial zero dimensions), this combination (A2.6) is identically zero. In this case we obtain one additional relation between quartic invariants. This happens for SU(2) and SU(3) with $\pi=+1$.

(c) There is also one bilinear combination of the initial tensor T

$$T^{\alpha\lambda}_{\mu\nu}T^{\mu\nu}_{\beta\lambda} - N^{-1}Q\delta^{\alpha}_{\beta} \tag{A2.7}$$

which transforms as an adjoint representation. If the adjoint representation does not appear in the symmetric part of $(\pi\times\pi)$ Kronecker product, the above combination (A2.7) vanishes identically. In this case we obtain again one new relation between quartic invariants. This happens for SU(2), $\pi=+1$ and for SU(4), $\pi=-1$.

(d) Finally, there is one relation which is a direct consequence of the invariant Levi-Civita tensor (ε symbol); namely, starting from bilinear combination (of the initial tensor) with four contra- and four covariant indices we can construct, using ε tensors, a new self-dual tensor with less than four co- and four contravariant indices. In this case we obtain again one new relation between quartic invariants. This happens when $N<8$ for $\pi=-1$, and when $N<4$ for $\pi=+1$.

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